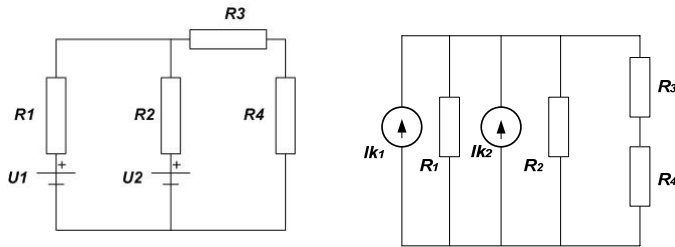


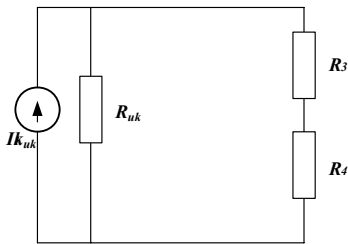
13. Odredi kolika se snaga troši na otporu R_4 ako su zadani elementi mreže spojeni prema slici: $U_1=U_2=12V, R_1=0,2\Omega, R_2=0,3\Omega, R_3=0,1\Omega, R_4=0,2\Omega$.



$$Ik_1 = \frac{U_1}{R_1} = 60 A$$

$$Ik_2 = \frac{U_2}{R_2} = 40 A$$

$$R_{uk} = \frac{R_1 \cdot R_2}{R_1 + R_2} = 0,12\Omega$$



$$I_{uk} = Ik_1 + Ik_2 = 100 A$$

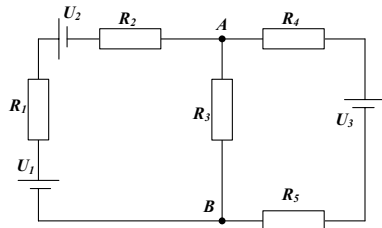
$$U_{uk} = I_{uk} \cdot R_{uk} = 100 \cdot 0,12 = 12 V$$

$$R_{34} = R_3 + R_4 = 0,3\Omega$$

$$I_4 = \frac{U_{uk}}{R_{uk} + R_3 + R_4} = 28,57 A$$

$$P_4 = I_4^2 \cdot R_4 = 163,26 W$$

14. Zadana je mreža prema slici. Odredite struju I_3 kroz R_3 ako su vrijednosti $U_1=64V, U_2=24V, U_3=30V, R_1=2\Omega, R_2=3\Omega, R_3=4\Omega, R_4=7\Omega$ i $R_5=3\Omega$.



$$I = \frac{U_{AB}}{R_3}$$

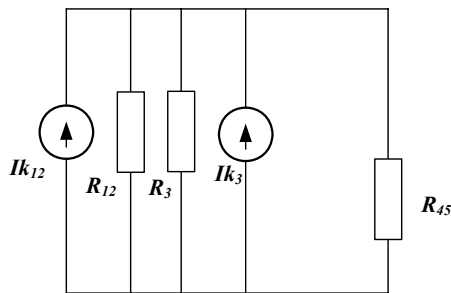
$$U_{12} = U_1 - U_2 = 64 - 24 = 40 V$$

$$R_{12} = R_1 + R_2 = 2 + 3 = 5\Omega$$

$$R_{45} = R_4 + R_5 = 10\Omega$$

$$Ik_{12} = \frac{U_{12}}{R_{12}} = 8 A$$

$$Ik_3 = \frac{U_3}{R_{45}} = 3 A$$



Nastavak zadatka 14.

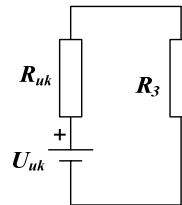
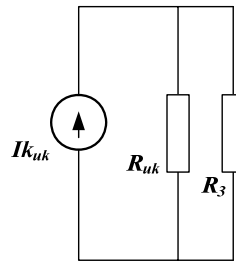
$$I_{uk} = I_{12} + I_3 = 11A$$

$$\frac{1}{R_{uk}} = \frac{1}{R_{12}} + \frac{1}{R_{45}} = R_{uk} = 3.33\Omega$$

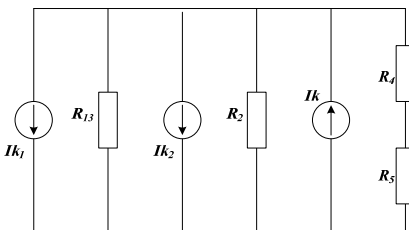
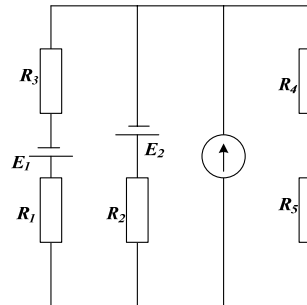
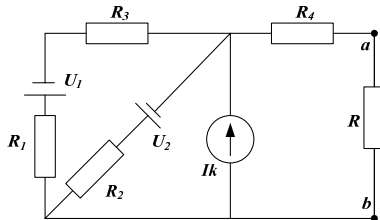
$$U_{uk} = I_{uk} \cdot R_{uk} = 36.63V$$

$$I(R_{uk} + R_3) = U_{uk}$$

$$I = 5A$$



15. U mreži prema slici odredite struju I_R kroz otpornik R i snagu koja se na njemu troši. Zadani su elementi u mreži: $U_1=12V$, $U_2=12V$, $I_k=14A$, $R_1=5\Omega$, $R_2=4\Omega$, $R_3=1\Omega$, $R_4=2\Omega$ i $R=10\Omega$.



$$I_{k1} = \frac{E_1}{R_1 + R_3} = 2A$$

$$I_{k2} = \frac{E_2}{R_2} = 3A$$

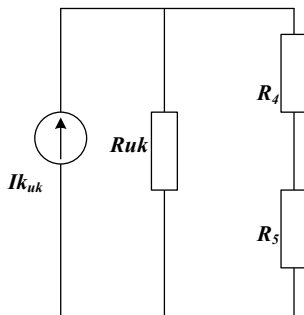
$$I_{kuk} = I_{k1} + I_{k2} - I_k = 13A$$

$$\frac{1}{R_{uk}} = \frac{1}{R_{13}} + \frac{1}{R_2} = 2.4A$$

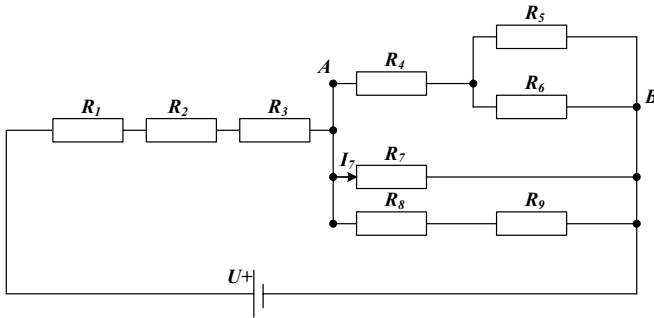
$$E_{uk} = I_{kuk} \cdot R_{uk} = 31.2V$$

$$I = \frac{E_{uk}}{R_{uk} + R_4 + R_5} = 2.16A$$

$$P = I^2 \cdot R_5 = 46.8W$$



16. U mreži prema slici potrebno je izračunati snagu P na otporniku R_9 , te ukupnu struju I i ukupan napon U spoja. Zadana je struja I_7 kroz otpornik R_7 od 4A, te sljedeće vrijednosti otpornika: $R_1=R_2=R_3=1\Omega$, $R_5=R_6=2\Omega$, $R_4=3\Omega$, $R_7=4\Omega$, $R_8=R_9=5\Omega$.



$$U_7 = U_{AB}$$

$$U_7 = I_7 \cdot R_7 = 16V$$

$$R_{89} = R_8 + R_9 = 10\Omega$$

$$I_{89} = \frac{U_{AB}}{R_{89}} = 1.6A$$

$$I_{789} = I_7 + I_{89} = 5.6A$$

$$\frac{1}{R_{56}} = \frac{1}{R_5} + \frac{1}{R_6} = 1\Omega$$

$$R_{456} = R_4 + R_{56} = 4\Omega$$

$$I_{456} = \frac{U_{AB}}{R_{456}} = 4A$$

$$R_{123} = R_1 + R_2 + R_3 = 3\Omega$$

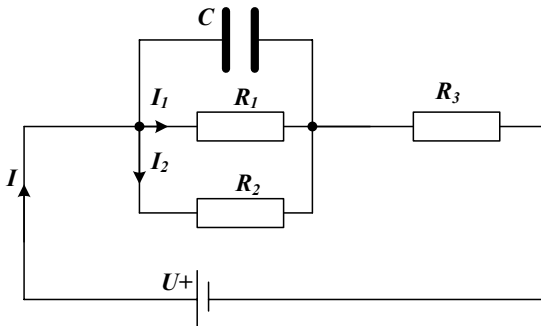
$$I_{uk} = I_{456} + I_{789} = 9.6A$$

$$U_{123} = I_{uk} \cdot R_{123} = 28.8V$$

$$U_{uk} = U_{123} + U_{AB} = 28.8 + 16 = 44.8V$$

$$P_9 = I_{89}^2 \cdot R_9 = 12.8W$$

16. Dva otpornika R_1 i R_2 od 25Ω i jedan otpornik $R_3=50\Omega$, te kondenzator kapaciteta $C=5\mu F$ spojeni su na izvor elektromotorne sile E prema slici. Odredite EMS izvora ako je naboj na kapacitetu $Q=200\mu C$.



II. Kirchoffov – zakon

$$E - U_{AB} - U_{BC} = 0$$

$$U_{AB} = U_C = U_{R1} = U_{R2} = \frac{Q}{C} = 40V$$

$$U_{BC} = U_{R3} = I \cdot R_3$$

I. Kirchoffov – zakon

$$I - I_1 - I_2 = 0$$

$$U_{R1} = U_{R2} = I_1 \cdot R_1 = I_2 \cdot R_2$$

$$I_1 = I_2 = \frac{40}{25} = 1.6A$$

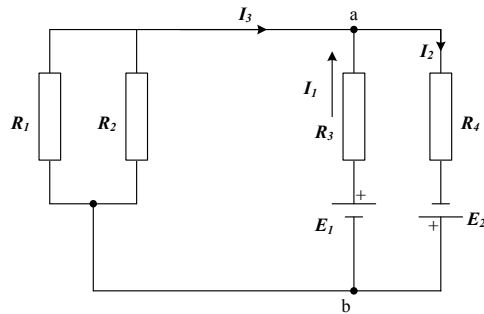
$$I = I_1 + I_2 = 3.2A$$

$$U_{BC} = I \cdot R_3 = 160V$$

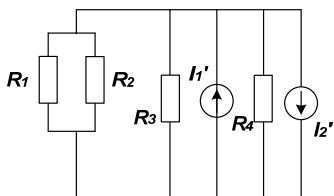
$$E = U_{AB} + U_{BC} = 40 + 160 = 200V$$

17. Zadane su vrijednosti za naponе izvora i otpore spojene prema shemi na slici ($U_1=150V$, $U_2=120V$, $R_1=4\Omega$, $R_2=12\Omega$, $R_3=6\Omega$, $R_4=3\Omega$). Treba odrediti:

- a) struje grane (I_1, I_2, I_3) i
b) razvijene snage na radnim otporima.

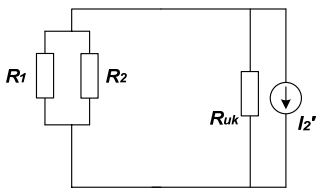


Kako bi odredili struje grana, treba naći napon između a i b.
Nadomjesna shema za paralelni spoj izvora:



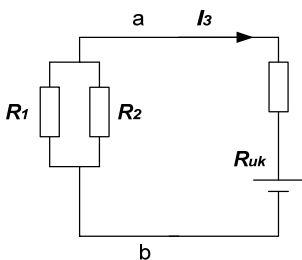
$$I_1' = \frac{U_1}{R_3} = 25 A$$

$$I_2' = \frac{U_2}{R_4} = 40 A$$

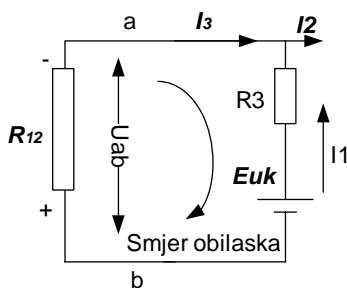


$$R_{34} = \frac{R_3 \cdot R_4}{R_3 + R_4} = 2 \Omega$$

$$I_{uk} = I_1' - I_2' = 15 A$$



$$U_{uk} = I_{uk} \cdot R_{uk} = 15 A \cdot 2 \Omega = 30 A$$



$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} = 3 \Omega$$

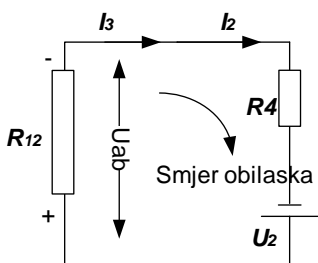
$$I_3 = \frac{U_{uk}}{R_{12} + R_{34}} = 6 A$$

$$U_{ab} = -I_3 \cdot R_{12} = -18 V$$

$$-U_1 + I_1 \cdot R_1 - I_3 \cdot R_{12} = 0$$

$$I_1 \cdot R_1 = U_1 + I_3 \cdot R_{12}$$

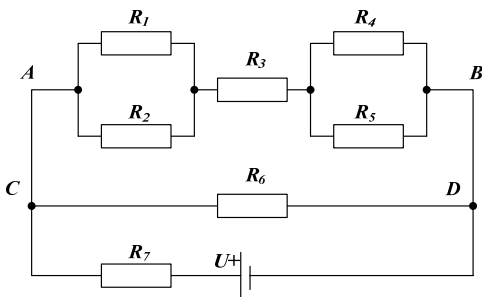
$$I_1 = \frac{U_1 + I_3 \cdot R_{12}}{R_3} = 28 A$$



$$U_2 - I_2 \cdot R_4 - I_3 \cdot R_{12} = 0$$

$$I_2 = \frac{U_2 + I_3 \cdot R_{12}}{R_4} = 34 A$$

18. Za spoj prema slici odredite napone na svim otporima. Zadano je: $R_1=R_7=4\Omega$, $R_2=12\Omega$, $R_3=7\Omega$, $R_4=R_5=20\Omega$, $R_6=80\Omega$ i $U=200V$.



$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{4 \cdot 12}{4 + 12} = 3\Omega$$

$$R_{45} = \frac{R_4 \cdot R_5}{R_4 + R_5} = \frac{20 \cdot 20}{20 + 20} = 10\Omega$$

$$R_{AB} = R_{12} + R_{45} + R_3 = 20\Omega$$

$$R_{AB,6} = \frac{20 \cdot 80}{20 + 80} = 16\Omega$$

$$R_{uk} = 16 + 4 = 20\Omega$$

$$I_{uk} = \frac{U_{uk}}{R_{uk}} = 10A$$

$$U_7 = I_{uk} \cdot R_7 = 40V$$

$$U_{AB} = U_6 = U - U_7 = 160V$$

$$I_{AB} = \frac{U_{AB}}{R_{AB}} = 8A$$

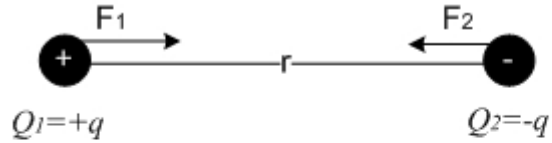
$$U_{12} = 8 \cdot 3 = 24V$$

$$U_3 = 8 \cdot 7 = 56V$$

$$U_{45} = 8 \cdot 10 = 80V$$

Zadaci prije kolokvija

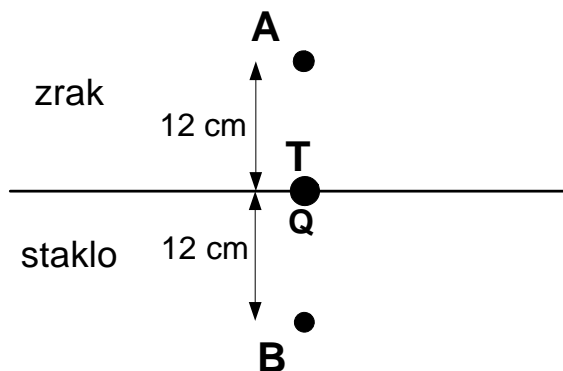
1. Dvije nabijene kugle u vakuumu se privlače silom F . Iznosi naboja na kuglama su jednaki. Ako s jedne kugle uzmemo $2/3$ njezinog naboja i premjestimo ga na drugu kuglu, kolika je sada privlačna sila?



$$|F_{12}| = \frac{1}{4\pi \cdot \epsilon_0 \cdot \epsilon_r} \cdot \frac{Q_1 \cdot Q_2}{r^2} = \frac{1}{4\pi \cdot \epsilon_0 \cdot \epsilon_r} \cdot \frac{q^2}{r^2}$$

$$|F_{12}'| = \frac{1}{4\pi \cdot \epsilon_0 \cdot \epsilon_r} \cdot \frac{\frac{1}{3}q \cdot \left(-\frac{1}{3}\right)q}{r^2} = \frac{1}{9} \cdot \frac{1}{4\pi \cdot \epsilon_0 \cdot \epsilon_r} \cdot \frac{q^2}{r^2} = \frac{1}{9} |F_{12}|$$

2. Točkasti naboj $Q=6\text{nC}$ nalazi se na graničnoj površini zrak-staklo čija je relativna permitivnost $\epsilon_r = 7$. Kolika je jakost električnog polja i električnog potencijala u točki A u zraku na udaljenosti $d=12\text{ cm}$ od točkastog naboja, a koliko u točki B u staklu? (Obavezno nacrtati sliku).



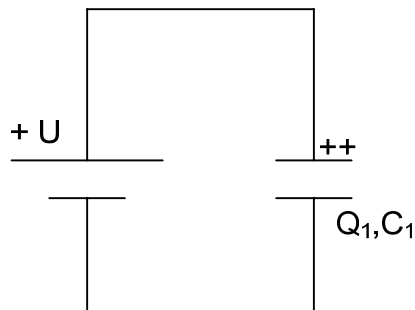
$$E_A = \frac{1}{4 \cdot \pi \cdot \epsilon_0 \cdot \epsilon_R} \cdot \frac{q}{r^2} = 4.5 \text{ V/m}$$

$$\varphi_A = \frac{1}{4 \cdot \pi \cdot \epsilon_0 \cdot \epsilon_R} \cdot \frac{q}{r} = 450 \text{ V}$$

$$E_B = \frac{1}{4 \cdot \pi \cdot \epsilon_0 \cdot \epsilon_R} \cdot \frac{q}{r^2} = 0.642 \text{ V/m}$$

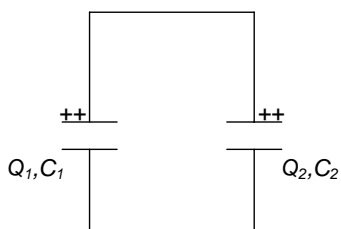
$$\varphi_B = \frac{1}{4 \cdot \pi \cdot \epsilon_0 \cdot \epsilon_R} \cdot \frac{q}{r} = 0.665 \text{ V}$$

3. Kondenzator kapaciteta $C_1=6\mu\text{F}$ nabija se na napon 220V. Isključivši C_1 s izvora, na njega je priključen prazan kondenzator nepoznatog kapaciteta koji se pri tome nabio na 22V. Koliki je iznos C_2 ?



$$Q = U \cdot C_1 = 220\text{V} \cdot 6\mu\text{F} = 1320\mu\text{C}$$

$$U = 220\text{V}$$



Dio naboja će preći sa C_1 na C_2 sve dok se naponi ne izjednače.

$Q_1 + Q_2 = Q$ – jer nismo doveli nove naboje!

$$U' = U_1 = U_2 = 22\text{V}$$

$$Q_1 = U_1 \cdot C_1 = 22\text{V} \cdot 3\mu\text{F} = 66\mu\text{C}$$

$$Q_2 = Q - Q_1 = 1320\mu\text{C} - 66\mu\text{C} = 1254\mu\text{C}$$

$$C_2 = \frac{Q_2}{U_2} = 57\mu\text{F}$$

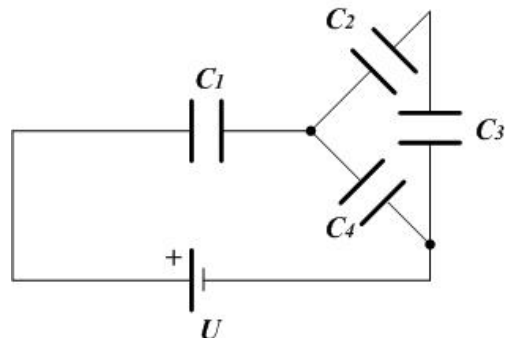
4. Odredite koliki je kapacitet kondenzatora C_3 u spoju na slici. Napon izvora je $U=15V$, napon na kondenzatoru $C_4=5 \cdot 10^{-6}F$ je $U_{C_4}=5V$, a naboj na kondenzatoru $C_2=7 \cdot 10^{-6}F$ iznosi $Q_2=25 \cdot 10^{-6}C$. Kapacitet kondenzatora $C_1=5 \cdot 10^{-6}F$.

$$Q_2 = Q_3 = 25 \cdot 10^{-6} C$$

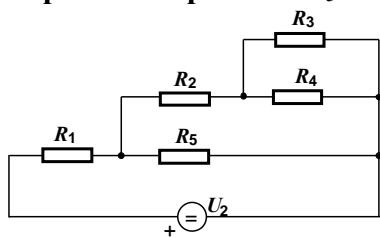
$$U_2 = \frac{Q_2}{C_2} = 3.57V$$

$$U_2 + U_3 = U_4 \Rightarrow U_3 = 1.43V$$

$$C_3 = \frac{Q_3}{U_3} = 1.74 \cdot 10^{-5} F$$



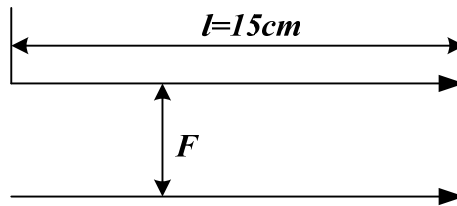
5. Zadan je spoj prema slici uz $R_1 = 2 \Omega$, $R_2 = 3 \Omega$, $R_3 = 9 \Omega$, $R_4 = 18 \Omega$ i $R_5 = 3 \Omega$, te pad napona na otporniku R_3 od $39 V$. Odrediti napon U_2 ?



6. Kad kroz akumulator teče struja jakosti 10 A napon na stezaljkama je 12V, dok pri struji od 25A napon je 11,4V. Koliki je unutarnji otpor i elektromotorna sila akumulatora?

Magnetizam

1. Kroz dva paralelna vodiča duljine 15m teče struja $I=60A$. Kolika mora biti njihova međusobna udaljenost, da oni jedan na drugi djeluju silom od 0.5N.



$$F = \mu_0 \cdot \frac{I_1 \cdot I_2 \cdot l}{2\pi r} = \mu_0 \frac{I_2 \cdot l}{2\pi r} \Rightarrow r = \frac{\mu_0 I_2 \cdot l}{2\pi F} = 0.0216 \text{ m}$$

2. Ravan vodič duljine 2m, mase 15g, smješten je u zraku okomito na magnetsko polje jakosti $5 \cdot 10^3 \text{ A/m}$. Nađite jakost struje koju treba propustiti vodič da on u magnetskom polju miruje.

Uvjet mirovanja: $F=G$

$$B \cdot I \cdot l \cdot \sin \alpha = m \cdot g$$

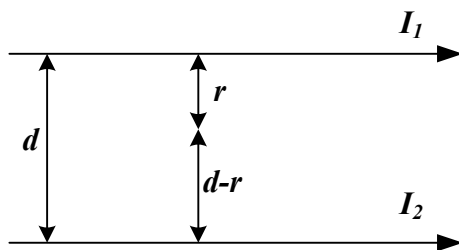
$$\sin \alpha = 1$$

$$B = \mu_0 \cdot H$$

$$\mu_0 \cdot H \cdot I \cdot l = m \cdot g$$

$$I = \frac{m \cdot g}{\mu_0 \cdot H \cdot l} = 11.7 \text{ A}$$

3. Kroz dva dugačka paralelna pravocrtna vodiča udaljena 20cm teku struje $I_1=4A$ i $I_2=6A$. na kojoj udaljenost od prvog vodiča iščezava magnetsko polje?

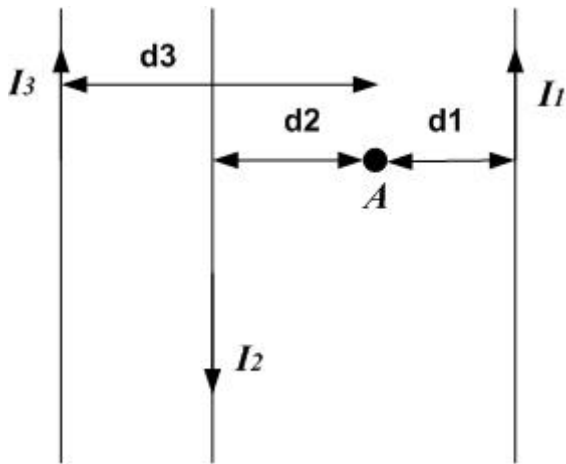


$$H = \frac{I}{2\pi r}$$

$$\frac{I_1}{2\pi r} = \frac{I_2}{2\pi(d-r)}$$

$$\frac{4}{r} = \frac{6}{20-r} = r = 8 \text{ cm}$$

4. Tri vodiča prema slici protjecana su strujama. Odrediti jakost magnetskog polja u točki A, ako je zadano: $I_1=10\text{A}$, $I_2=5\text{A}$, $I_3=5\text{A}$, $d_1=22,5\text{cm}$, $d_2=22,5\text{cm}$, $d_3=52,5\text{cm}$.



\vec{H} - vektor

$$\vec{H} = \vec{H}_1 + \vec{H}_2 - \vec{H}_3 = 9.08 \text{ A/m}$$

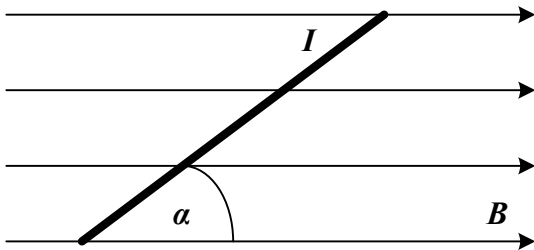
$$\vec{H}_1 = \frac{I_1}{2\pi d_1} = 7.07 \text{ A/m}$$

$$\vec{H}_2 = \frac{I_2}{2\pi d_2} = 3.53 \text{ A/m}$$

$$\vec{H}_3 = \frac{I_3}{2\pi d_3} = 1.52 \text{ A/m}$$

5. Kroz horizontalan štap duljine 0,2m prolazi el. struja. Štap se nalazi na horizontalnom magnetnom polju od 0,08T, koje sa smjerom struje zatvara kut od 30° . Sila kojom polje djeluje na štap iznosi 0,24N.

- koliki je iznos el. struje koji prolazi kroz štap?
- kolika bi sila bila da el. struja ima smjer magnetskog polja?



a)

$$F = B \cdot I \cdot l \cdot \sin \alpha$$

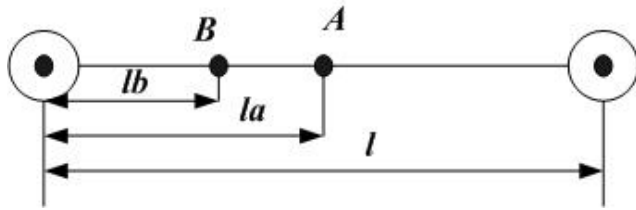
$$I = \frac{F}{B \cdot l \cdot \sin \alpha} =$$

b)

$$\sin \alpha = 0^\circ$$

$$F = B \cdot I \cdot l \cdot \sin \alpha =$$

6. Zadan je raspored vodiča prema slici. Razmak vodiča je $l=50\text{cm}$, a struje kroz vodiče su $I_1=50\text{A}$ i $I_2=100\text{A}$. treba odrediti jakost polja H u točkama A i B.



$$H = \frac{1}{2\pi r}$$

$$H_A = H_{A1} - H_{A2} = -31,89 \text{ A/m}$$

$$H_{A1} = \frac{I_1}{2\pi d_A} = 31,89 \text{ A/m}$$

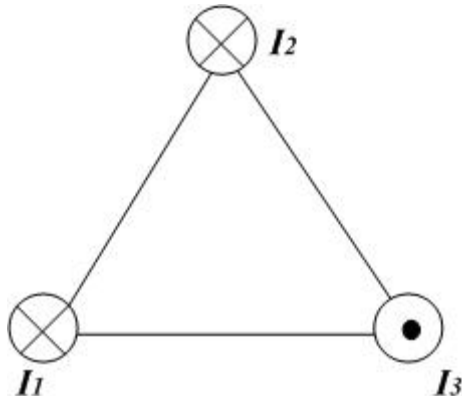
$$H_{A2} = \frac{I_2}{2\pi(l-l_A)} = 63,69 \text{ A/m}$$

$$H_B = H_{B1} - H_{B2} = 39,79 \text{ A/m}$$

$$H_{B1} = \frac{I_1}{2\pi d_B} = 79,57 \text{ A/m}$$

$$H_{B2} = \frac{I_2}{2\pi(l-l_B)} = 39,78 \text{ A/m}$$

7. Kroz tri paralelna vodiča protječu struje $I_1=10\text{A}$, $I_2=5\text{A}$, $I_3=15\text{A}$. Odredite smjer i iznos (po jedinici duljine) sila na vodiče ako presjeci vodiča čine jednakostraničan trokut stranice 1m ?



$$F_{12} = F_{21} = \mu_0 \frac{I_1 I_2}{2\pi a} \cdot l$$

$$F_{13} = F_{31} = \mu_0 \frac{I_1 I_3}{2\pi a} \cdot l$$

$$F_{23} = F_{32} = \mu_0 \frac{I_2 I_3}{2\pi a} \cdot l$$

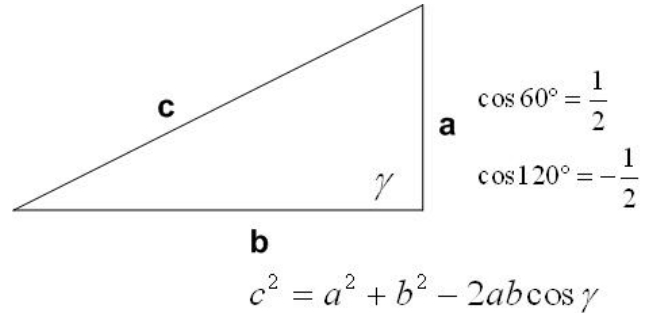
Nastavak 7. zadatka

Po jedinici duljine:

$$\frac{F_{12}}{l} = \mu_0 \frac{I_1 I_2}{2\pi a} = 10 \frac{\mu\text{N}}{\text{m}}$$

$$\frac{F_{13}}{l} = \mu_0 \frac{I_1 I_3}{2\pi a} = 30 \frac{\mu\text{N}}{\text{m}}$$

$$\frac{F_{23}}{l} = \mu_0 \frac{I_2 I_3}{2\pi a} = 15 \frac{\mu\text{N}}{\text{m}}$$



$$F_1^2 = F_{12}^2 + F_{13}^2 - 2F_{12}F_{13} \cos 60^\circ = \frac{F_1}{l} = 26,45 \mu\text{N} / \text{m}$$

$$F_2^2 = F_{12}^2 + F_{23}^2 - 2F_{12}F_{23} \cos 60^\circ = \frac{F_2}{l} = 13,29 \mu\text{N} / \text{m}$$

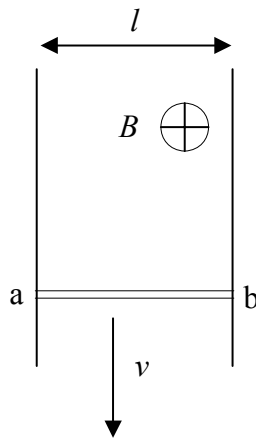
$$F_3^2 = F_{13}^2 + F_{23}^2 - 2F_{13}F_{23} \cos 120^\circ = \frac{F_3}{l} = 39,68 \mu\text{N} / \text{m}$$

8. Zavojnica induktiviteta $L=2\text{H}$, duljine $l=10\text{ cm}$ s brojem zavoja $N=500$ protjecana je strujom od 2A .

- a) Koliki se napon inducira u zavojnici ako se u vremenu od $0,01\text{ s}$ struja poveća na 7 A .
- b) Kolika je jakost magnetskog polja H u jezgri zavojnice.

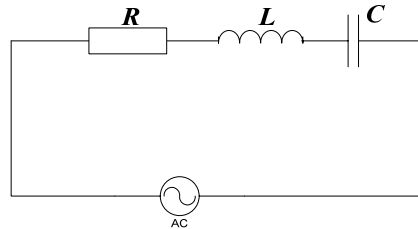
Zavojnica duljine 1m sa 1000 zavoja sa jezgrom presjeka 5cm^2 od transformatorskog lima kod struje 1A ima indukciju $0,8\text{T}$. Odrediti relativnu permabilnost jezgre?

10. U homogenom magnetskom polju indukcije $B=300\text{T}$ u vertikalnoj ravnini su postavljene tračnice od kovine na rastojanju $l=500\text{ cm}$. Tračnice su na vrhu spojene. Pravac vektora magnetske indukcije je normalan na ravan koju obrazuju tračnice. Niz tračnice klizi bez trenja (ali u stalnom kontaktu sa tračnicama) prav vodič duljine $ab=500\text{ cm}$, stalnom brzinom $v=5\text{m/s}$. Masa vodiča ab je $m=1\text{ g}$. Odrediti otpor pokretnog vodiča ako je otpor ostalih vodiča zanemariv.



Izmjenična struja

1. Serijski spoj otpornika otpora $R=2\Omega$, zavojnice induktiviteta $L=1.6\text{mH}$ i kondenzator kapaciteta $C=20\mu\text{F}$ priključen je na izvor izmjeničnog napona efektivne vrijednosti $U=6\text{V}$, kružne frekvencije $\omega=5000\text{ rad/s}$. Odrediti struju u krugu te padove napona na pojedinim elementima kruga.



$$X_L = \omega \cdot L = 5000 \cdot 1.6 \cdot 10^{-3} = 8\Omega$$

$$X_C = \frac{1}{\omega \cdot C} = \frac{1}{5000 \cdot 20 \cdot 10^{-6}} = 10\Omega$$

Impedancija kruga:

$$Z = R + j(X_L - X_C) = 2 + j(8 - 10) = 2 - j2 = 2\sqrt{2} \angle -45^\circ \Omega$$

jer je:

$$\varphi = \arctg\left(\frac{-2}{2}\right) = \arctg(-1) = -45^\circ$$

Fazor struje:

$$\vec{I} = \frac{\vec{U}}{\vec{Z}} = \frac{6 \angle 0^\circ}{2\sqrt{2} \angle -45^\circ} = 1.5\sqrt{2} \angle 45^\circ \text{ A}$$

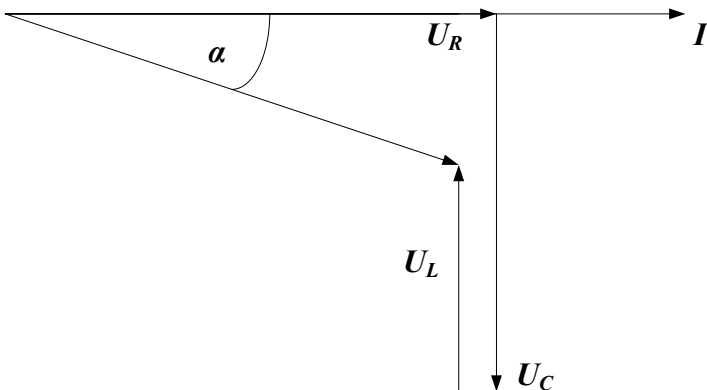
Padovi napona:

$$\vec{U}_R = \vec{I} \cdot R = 1.5\sqrt{2} \angle 45^\circ \cdot 2 = 4.24 \angle 45^\circ [\text{V}]$$

$$\vec{U}_L = \vec{I} \cdot jX_L = 1.5\sqrt{2} \angle 45^\circ \cdot 8 \angle 90^\circ = 12\sqrt{2} \angle 135^\circ [\text{V}]$$

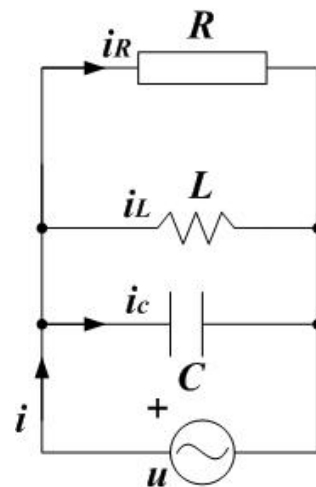
$$\vec{U}_C = \vec{I} \cdot (-jX_C) = 1.5\sqrt{2} \angle 45^\circ \cdot 10 \angle -90^\circ = 15\sqrt{2} \angle -45^\circ [\text{V}]$$

Vektorski dijagram:



2. Na izmjenični izvor efektivnog napona 10V i kružne frekvencije 1000Hz paralelno priključimo otpor $R=4\Omega$, kondenzator $C=0,025\text{mF}$ i induktivitet $L=8\text{mH}$.

- Izračunaj impedanciju priključenu na izvor
- Kolika je efektivna struja i kolike su njene komponente



a)

$$R = 4\Omega$$

$$\vec{X}_L = jX_L = j8\Omega$$

$$\vec{X}_C = -j4\Omega$$

$$\frac{1}{\vec{Z}} = \frac{1}{R} + \frac{1}{\vec{X}_L} + \frac{1}{\vec{X}_C} \Rightarrow \vec{Z} = 3.2 - 1.6j \Rightarrow \vec{Z} = 3.57 \angle -26.57^\circ \Omega$$

b)

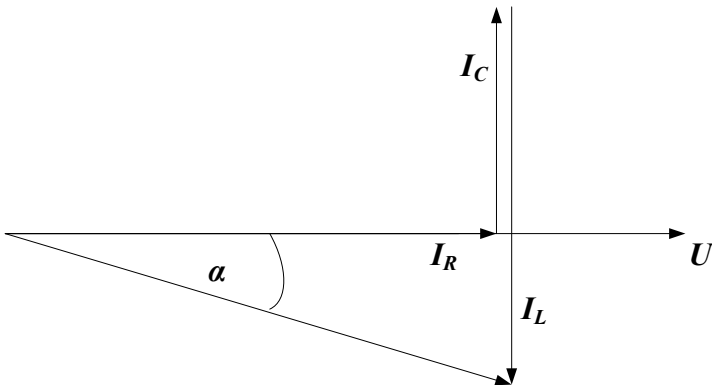
$$I_{ef} = \frac{U_{ef}}{Z_{ef}} = \frac{10 \angle 0^\circ}{3.57 \angle -26.57^\circ} = 2.79 \angle 26.57^\circ A$$

$$\vec{I}_R = \frac{U_{ef}}{R} = 2.5 A$$

$$\vec{I}_L = \frac{U_{ef}}{\vec{X}_L} = 1.25 \angle -90^\circ A$$

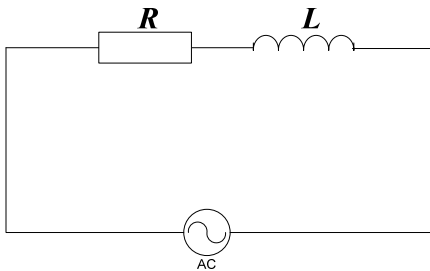
$$\vec{I}_C = \frac{U_{ef}}{\vec{X}_C} = 2.5 A \angle 90^\circ A$$

Vektorski dijagram:



3. U serijski strujni krug spojeni su otpor $R=4\ \Omega$, zavojnica $L=3\text{mH}$ i izmjenični naponski izvor $u=10\sqrt{2}\sin(1000t)$.

- Napiši izraz za struju.
- Koliki su padovi napona na otporu i induktivitetu?
- Skiciraj vektorski prikaz napona i struje



a)

$$X_L = \omega \cdot L = 3\ \Omega$$

$$\operatorname{tg} \varphi = \frac{3}{4} \Rightarrow \varphi = 36.87^\circ$$

$$Z = \sqrt{R^2 + X_L^2} = 5\ \Omega$$

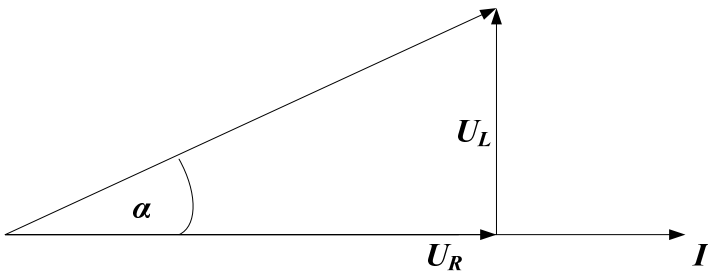
$$\bar{I} = \frac{\bar{U}}{Z} = \frac{10}{5} = 2\ \text{A}$$

b)

$$\bar{U}_R = \bar{I} \cdot R = 2 \angle -36.87^\circ \cdot 4 = 8[\text{V}]$$

$$\bar{U}_L = \bar{I} \cdot jX_L = 2 \angle -36.87^\circ \cdot 3 \angle 90^\circ = 6 \angle 53.13^\circ [\text{V}]$$

Vektorski dijagram:



4. Efektivni pad napona na kombinaciji elemenata L , R_1 i C (između točaka A i C) je $U_I=10V$. Ako je impedancija zavojnice jednaka $X_L=10\Omega$, a impedancija kondenzatora $X_C=5\Omega$, koliki je efektivni pad napona između točaka B i D? Vrijednosti otpornika su $R_1=3\Omega$ i $R_2=4\Omega$.

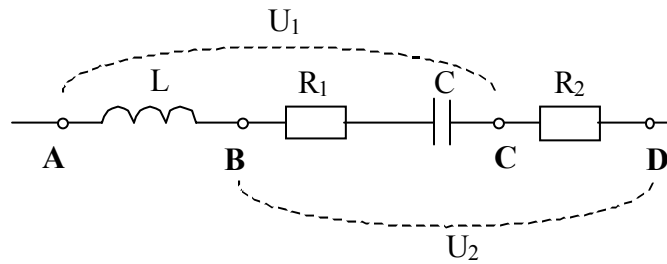
$$X_L = 10\Omega$$

$$X_C = 5\Omega$$

$$U_{AC} = 10V$$

$$R_1 = 3\Omega$$

$$R_2 = 4\Omega$$



$$Z_{AC}^2 = R^2 + (X_L - X_C)^2$$

$$Z_{AC} = 5.83\Omega$$

$$Z_{BC}^2 = R_{12}^2 + X_C^2$$

$$Z_{BC} = 8.60\Omega$$

$$I = \frac{U_{AC}}{Z_{AC}} = 1.71A$$

$$U_{BC} = I \cdot Z_{BC} = 14.75V$$

5. Na serijski spoj omskog i jalovog otpora priključena je napon $u = 220\sqrt{2} \sin 100\pi t$ V.

Krugom teče struja $i = 2 \sin\left(100\pi + \frac{\pi}{6}\right)t$ A.

- Kolika je efektivna vrijednost struje
- Odredite faktor snage
- Kolike su vrijednosti elemenata
- Kolika se energija pretvara u toplinu za 1 minutu?

$$\text{a) } I_{ef} = \frac{I_m}{\sqrt{2}} = \frac{2}{\sqrt{2}} = 1.41A$$

$$\text{b) } \varphi = \frac{\pi}{6} \Rightarrow \cos \varphi = \frac{\sqrt{3}}{2} = 0.866$$

$$\text{c) } U = \frac{U_m}{\sqrt{2}} = \frac{220\sqrt{2}}{\sqrt{2}} = 220V$$

$$U_R = U \cdot \cos \rho = 220 \cdot 0.866 = 190.5V$$

$$U_C = U \cdot \sin \rho = 220 \cdot 0.5 = 110V$$

$$R = \frac{U_R}{I} = \frac{190.5}{\sqrt{2}} = 134.7\Omega$$

$$X_C = \frac{U_C}{I} = \frac{110}{\sqrt{2}} = 77.8\Omega \Rightarrow C = \frac{1}{2\pi f \cdot X_C} = 40.9\mu F$$

$$W = U \cdot I \cdot \cos \varphi \cdot t = 220\sqrt{2} \cdot 0.866 \cdot 60 = 16.166J$$

6. Žarulja 220V/60W spojena je na napon $u = 220\sqrt{2} \sin 100\pi t$. Na raspolaganju su nam kondenzator $C=1\mu\text{F}$ i zavojnica $L=1\text{H}$.

- Da li je potrebno spojiti neki od elemenata sa žaruljom da bi ona svjetlila punim sjajem.
- Kolika je $i=f(t)$ ako u seriju sa žaruljom spojimo zavojnicu?
- Koliku struju mjeri ampermetar u slučaju b?
- Koliki mora biti iznos kapaciteta kondenzatora spojenog serijski sa zavojnicom u b. slučaju da bi žarulja svjetlila punim sjajem.

a) $U_m = 220\sqrt{2} \Rightarrow U_{ef} = 220V$

$$I = \frac{U}{R}; R_{\check{z}} = \frac{U_n^2}{P} = \frac{220^2}{60} = 806.67\Omega$$

$$I = \frac{U}{R} = \frac{220}{806.67} = 0.273A$$

$$P = I^2 \cdot R_{\check{z}} = 60W - \text{nije potrebno}$$

b) $X_L = 2\pi fL = 100\pi = 314\Omega$

$$Z = \sqrt{R^2 + X_L^2} = 865.6\Omega$$

$$\cos \varphi = \frac{R_{\check{z}}}{Z} = \frac{806.67}{865.6} = 0.93 \Rightarrow \varphi = 21.56^\circ$$

$$i(t) = 0.358 \cdot \sin(314t - 21.56^\circ)$$

c) $I_{ef} = \frac{I_m}{\sqrt{2}} = 0.253A$

d) $X_L = X_C \Rightarrow X_C = 314\Omega$

$$C = \frac{1}{2\pi f \cdot X_C} = 10.1\mu\text{F}$$

7. Jednofazno trošilo priključeno na 220V/50Hz ima radnu snagu 400W, a jalovu 200Var induktivnog karaktera.

- Koliki je iznos struje spoja?
- Koja je vrijednost faktora snage $\cos\varphi$?
- Ako je trošilo izvedeno kao serijski spoj otpora i idealne zavojnice, koju vrijednost ima otpor?

a)

$$P = U \cdot I \cdot \cos\varphi \Rightarrow I = \frac{P}{U \cdot \cos\varphi} = \frac{400}{220 \cdot \cos 26.57} = 2.03 A$$

$$\operatorname{tg}\varphi = \frac{P_Q}{P_R} = \frac{200}{400} = \frac{1}{2} \Rightarrow \varphi = 26.57^\circ$$

b)

$$\cos\varphi = \cos 26.57^\circ = 0.89$$

c)

$$P_S = U \cdot I = 220 \cdot 2.03 = 446.6 VA$$

$$P_{QL} = I_L \cdot U_L \Rightarrow U_L = \frac{P_{QL}}{I_L} = 98.5 V$$

$$U_L = I_L \cdot X_L \Rightarrow X_L = \frac{U_L}{I_L} = 48.5 \Omega$$

$$Z = \frac{U}{I} = \frac{220}{2.03} = 108.4 \Omega$$

$$R = \sqrt{Z^2 - XL^2} = 96.9 \Omega$$

8. U Europi je frekvencija gradske mreže 50 Hz, a u Americi 60Hz. Za koliko je posto induktivni otpor neke zavojnice induktiviteta L veći u Americi nego u EU?

$$X_L = \omega L = 2\pi f \cdot L$$

$$X_{L1} = \omega L = 100\pi \cdot L$$

$$X_{L2} = \omega L = 120\pi \cdot L$$

$$\frac{X_{L2}}{X_{L1}} = \frac{120\pi L}{100\pi L} = \frac{120}{100} = 1.2 \Rightarrow 20\%$$

9. Zavojnica nekog omskog otpora 10Ω i induktiviteta L priključena je na izmjenični napon $U=127V$ i $f=50Hz$. Zavojnica troši snagu $P=400W$, a fazni pomak između struje i napona je 60° .

- Koliki je induktivitet zavojnice?
- Kolika je struja kroz zavojnicu?
- Kolika je jalova snaga?



Prividna snaga

$$S = U \cdot I \Rightarrow VA$$

Radna (Djelatna) snaga – snaga na otporima

$$P = U \cdot I \cdot \cos \varphi \Rightarrow W$$

$$P = I^2 \cdot R = \frac{U^2}{R}$$

Reaktivan (Jalova) snaga

$$Q = U \cdot I \cdot \sin \varphi \Rightarrow VAr - Volt \text{ Amper reaktivni}$$

$$a) \operatorname{tg} \varphi = 60^\circ = \frac{I_m}{R} = \frac{\omega L}{R} \Rightarrow L = \frac{R \cdot \operatorname{tg} \varphi}{\omega} = 0,055 \text{H}$$

$$b) P = U \cdot I \cdot \cos \varphi \Rightarrow I = \frac{P}{U \cos \varphi} = 6,29 \text{A}$$

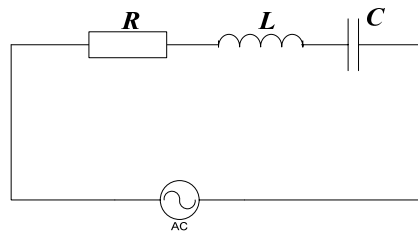
$$c) Q = U \cdot I \cdot \sin \varphi = 691 \text{VAr}$$

10. Serijski su spojeni $R=4\Omega$, $L=3\text{mH}$ i $C=166\mu\text{F}$ priključeni su na napon $u=100\sqrt{2}\sin 1000t$. Odredite:

a) ukupnu impedanciju i admitanciju spoja

b) struju spoja

c) napone na pojedinim elementima



$$a) X_L = \omega \cdot L = 3\Omega$$

$$X_C = \frac{1}{\omega C} = 6,02\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 5\Omega$$

$$Y = \frac{1}{Z} = 0,2\Omega$$

$$\cos \varphi \Rightarrow \varphi = -36,9^\circ$$

b)

$$U_{ef} = \frac{U_m}{\sqrt{2}} = 100 \text{V}$$

$$I = \frac{U_{ef}}{Z} = 20 \text{A}$$

$$i(t) = 20\sqrt{2} \sin(\omega t + 36,9^\circ)$$

c)

$$U_R = I \cdot R = 80 \text{V}$$

$$U_L = I \cdot X_L = 60 \angle 126,9^\circ \text{V}$$

$$U_C = I \cdot X_C = 120 \angle -53,1^\circ \text{V}$$